

Non-renormalization for planar Wess-Zumino model

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Abstract

Using a non-perturbative functional method, where the quantum fluctuations are gradually set up, it is shown that the interaction of a $N = 1$ Wess-Zumino model in 2+1 dimensions does not get renormalized. This result is valid in the framework of the gradient expansion and aims at compensating the lack of non-renormalization theorems.

Non-renormalization theorems are based on analyticity properties and are thus not present for $N = 1$ supersymmetry in 2+1 dimensions, since the odd coordinate of superspace is real. Supersymmetric properties should nevertheless restrict the quantum corrections in this situation, and lead to a control of the renormalization processes.

In the framework of relativistic-like effective descriptions of high-temperature superconductivity [1], supersymmetric models in 2+1 dimensions have been introduced [2], where conditions for the elevation to a $N = 2$ supersymmetry were studied, motivated by the presence of non-renormalization theorems in this case. It is though interesting to look at the possibility to have exact results with $N = 1$ supersymmetry and a functional method is presented here for this purpose, which gives indications on the renormalized structure of a Wess-Zumino model in 2+1 dimensions.

The idea of the method is to control quantum fluctuations with the mass of the bare theory. When this mass is very large, the quantum fluctuations are frozen and the system is classical. As the bare mass decreases, the quantum fluctuations appear and the parameters of the theory get dressed. We can then consider the "fluctuation flows" of the effective action (the proper graphs generator functional) with the bare mass and, starting from the bare action, follow these flows so as to built the "full" quantum theory, containing all the quantum effects.

There are similarities between this procedure and the blocking procedure [3], since the latter describes the evolution of a theory with a momentum, from the

ultraviolet (UV) scales to the infrared (IR) ones. The present method, though, is not based on a splitting the UV from the IR degrees of freedom and thus does not introduce any artificial coarse graining function. It was shown, in previous works [4, 5], that these fluctuation flows recover the usual one-loop results. Beyond one-loop, the results given by these flows do not coincide anymore with a loop expansion, since the results are based on the so-called gradient expansion. Note finally that this method does not require any regularization in 2+1 dimensions, which is another advantage to use it.

We will find, in the context of the gradient expansion, the exact effective action for a Q^4 Wess-Zumino theory (Q is the scalar superfield and Q^4 leads to the marginal interaction ϕ^6 , where ϕ is the scalar component of Q). We will see that only the mass gets renormalized, whereas the interaction does not.

We note $z = (x, \theta)$ the coordinate of superspace and we take the conventions used in [6]. The bare action is

$$S_\lambda[Q] = \int d^5z \left\{ \frac{1}{2} Q D^2 Q + \frac{\lambda}{2} m_B Q^2 + \frac{g_B}{24} Q^4 \right\}, \quad (1)$$

where g_B is the dimensionless bare coupling. The effective action Γ_λ , defined as the Legendre transform of the connected graphs generator functional, depends on λ : for $\lambda \gg 1$ Γ_λ describes the classical theory, and as λ decreases down to 1 the quantum theory emerges out of the increasing quantum fluctuations. The massless case can be obtained by letting λ decrease down to 0.

The exact and non-perturbative evolution equation of the effective action with the parameter λ was derived in [5] and reads

$$\partial_\lambda \Gamma_\lambda = \frac{m_B}{2} \text{Tr} \left\{ Q^2 + \left(\frac{\delta^2 \Gamma_\lambda}{\delta Q \delta Q} \right)^{-1} \right\}, \quad (2)$$

where the trace "Tr" is to be taken over the superspace coordinates. In the framework of the gradient expansion, we consider the following ansatz for the functional dependence of Γ :

$$\Gamma_\lambda[Q] = \int d^5z \left\{ \frac{1}{2} Q D^2 Q + U_\lambda(Q) \right\}, \quad (3)$$

i.e. we allow any potential term, but do not take into account higher order kinetic terms or derivative interactions. In this situation, in order to find the evolution of the potential, it is enough to consider a constant configuration Q_0 of the superfield Q . Q_0 is a vacuum expectation value of the scalar component of Q . We have in principle $\int dp^3 = \infty$ and $\int d^2\theta = 0$, but we consider the finite, regularized, volume of superspace

$$\mathcal{V} = \int \frac{dp^3}{(3\pi)^3} d^2\theta. \quad (4)$$

We have then for the effective action

$$\Gamma_\lambda[Q_0] = \mathcal{V}U_\lambda(Q_0), \quad (5)$$

and for its second derivative

$$\begin{aligned} & \frac{\delta^2 \Gamma_\lambda}{\delta Q(p, \theta) \delta Q(q, \theta')} \Big|_{Q=Q_0} \\ = & \left[U_\lambda^{(2)}(Q_0) + D_{p\theta}^2 \right] (2\pi)^3 \delta^3(p+q) \delta^2(\theta - \theta'), \end{aligned} \quad (6)$$

where $U_\lambda^{(2)}(Q_0)$ denotes the second derivative of the potential with respect to Q_0 . Using then the properties [6]

$$\begin{aligned} \delta^2(\theta - \theta') \delta^2(\theta' - \theta) &= 0 \\ \delta^2(\theta - \theta') D_{p\theta}^2 \delta^2(\theta' - \theta) &= \delta^2(\theta - \theta') \\ (D_{p\theta}^2)^2 &= -p^2, \end{aligned} \quad (7)$$

we easily obtain

$$\begin{aligned} & \left(\frac{\delta^2 \Gamma_\lambda}{\delta Q(p, \theta) \delta Q(q, \theta')} \Big|_{Q=Q_0} \right)^{-1} \\ = & \frac{U_\lambda^{(2)}(Q_0) - D_{p\theta}^2}{p^2 + [U_\lambda^{(2)}(Q_0)]^2} (2\pi)^3 \delta^3(p+q) \delta^2(\theta - \theta'), \end{aligned} \quad (8)$$

and then

$$\begin{aligned} & \text{Tr} \left(\frac{\delta^2 \Gamma_\lambda}{\delta Q \delta Q} \Big|_{Q=Q_0} \right)^{-1} \\ = & \int \frac{dp^3}{(2\pi)^3} \frac{dq^3}{(2\pi)^3} d^2\theta d^2\theta' \delta^3(p+q) \delta^2(\theta - \theta') \\ & \times \left(\frac{\delta^2 \Gamma_\lambda}{\delta Q(p, \theta) \delta Q(q, \theta')} \Big|_{Q=Q_0} \right)^{-1} \\ = & -\mathcal{V} \int \frac{dp^3}{(2\pi)^3} \frac{1}{p^2 + [U_\lambda^{(2)}(Q_0)]^2}. \end{aligned} \quad (9)$$

This last integral is divergent but can be written:

$$\begin{aligned}
& \text{Tr} \left(\frac{\delta^2 \Gamma_\lambda}{\delta Q \delta Q} \Big|_{Q=Q_0} \right)^{-1} \\
&= -\frac{\mathcal{V}}{2\pi^2} \int_0^\infty dp + \frac{\mathcal{V}}{2\pi^2} \int_0^\infty dp \frac{[U_\lambda^{(2)}(Q_0)]^2}{p^2 + [U_\lambda^{(2)}(Q_0)]^2} \\
&= \text{Const.} + \frac{\mathcal{V}}{4\pi} U_\lambda^{(2)}(Q_0),
\end{aligned} \tag{10}$$

where the diverging constant does not depend on Q_0 . Plugging this result in the evolution equation (2) and discarding the field-independent terms, we finally obtain

$$\partial_\lambda U_\lambda(Q_0) = \frac{m_B}{2} Q_0^2 + \frac{m_B}{8\pi} U_\lambda^{(2)}(Q_0). \tag{11}$$

Let us now turn to the solution of this equation. Starting from the initial interaction (1), we see that, when decreasing the fluctuation parameter from λ to $\lambda - \delta\lambda$, the potential $U_{\lambda-\delta\lambda}$ will not acquire higher powers of Q_0 than the ones contained in U_λ since

$$U_{\lambda-\delta\lambda}(Q_0) = U_\lambda(Q_0) - \delta\lambda \frac{m_B}{2} \left[Q_0^2 + \frac{1}{4\pi} U_\lambda^{(2)}(Q_0) \right] \tag{12}$$

As a consequence, we consider the following ansatz for $U_\lambda(Q_0)$:

$$U_\lambda(Q_0) = u_0(\lambda) + \frac{1}{2} u_1(\lambda) Q_0^2 + \frac{1}{24} u_2(\lambda) Q_0^4, \tag{13}$$

and plug it in the evolution equation (11), to find the exact solution

$$\begin{aligned}
u_2(\lambda) &= g \\
u_1(\lambda) &= \lambda m_B \left(1 + \frac{g}{8\pi} \right) + M \\
u_0(\lambda) &= \frac{m_B}{8\pi} \left[\frac{\lambda^2}{2} m_B \left(1 + \frac{g}{8\pi} \right) + M\lambda + a \right],
\end{aligned} \tag{14}$$

where (g, M, a) are constants of integration to be determined with initial conditions. To determine g , we invoke the central idea of this method, i.e. the fact that for $\lambda \rightarrow \infty$, the theory should be the classical one and thus $g = g_B$. The choice of M must be made with another boundary condition, since $u_1(\lambda)$ diverges as $\lambda \rightarrow \infty$. With an initial condition such that $\lambda \gg 1$, we should take a *finite* value of λ , and therefore also an initial effective action which would already contain some quantum effects. To avoid this, we take the boundary condition at $\lambda = 0$, where the bare theory is massless. There the bare action S

does not contain any dimensionfull parameter, and also no mass parameter is introduced for any regularization, such that no dynamical mass can be generated in the quantum theory. We conclude that $M = 0$. Finally, the constant a is not important since it deals with field-independent terms.

Discarding the field-independent terms, the λ -dependent effective theory is eventually described by the following effective action

$$\Gamma_\lambda[Q] = \int d^5z \left\{ \frac{1}{2} Q D^2 Q + \frac{\lambda}{2} m_B \left(1 + \frac{g_B}{8\pi} \right) Q^2 + \frac{g_B}{24} Q^4 \right\}. \quad (15)$$

To conclude, we see that no new interaction is generated by the quantum fluctuations and the bare interaction does not get any quantum correction, but only the mass term does. The full quantum theory is then described by the effective action

$$\Gamma_{\lambda=1}[Q] = \int d^5z \left\{ \frac{1}{2} Q D^2 Q + \frac{m_B}{2} \left(1 + \frac{g_B}{8\pi} \right) Q^2 + \frac{g_B}{24} Q^4 \right\}, \quad (16)$$

which is exact in the framework of the gradient expansion (3). This is consistent with [5], where the potential was truncated to the order of bare interaction.

We also see that the massless theory ($\lambda \rightarrow 0$) does not get any quantum correction at all, in the approximation (3), since

$$\Gamma_{\lambda=0}[Q] = \int d^5z \left\{ \frac{1}{2} Q D^2 Q + \frac{g_B}{24} Q^4 \right\} = S_{\lambda=0}[Q]. \quad (17)$$

This result should not be confused, yet, with a non-renormalization theorem: higher order derivative terms or derivative interactions would lead to a renormalization of the potential and this study is let for a future work. But, as expected, the supersymmetric structure of the theory can still lead to strong predictions concerning the quantum corrections.

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